

Seminar “Gradient Flows”

Ben Schweizer, TU Dortmund, Summer 2024

6.3.2024

Themes

- G**- Geometry; Onsager operator
- M**- Metric setting: Minimizing movements
- W**- Wasserstein as metric space
- C**- Curves in probability space, continuity equation

Talks

1. Gradient flows and differential geometry, a first look at [Ott01]. (B.S.)
2. Gradient flows in Hilbert spaces, [Mie23, 1-2] (Differentials, Minimization scheme, existence result). (M.O.)
3. Banach space setting, [Mie23, 3] (Legendre duality, chain rule, existence result). (F.L.)
4. Metric space setting I, [AGS08, 1], [Mie23, 4] (Upper gradients, maximal slope). Aim: Understand equivalences. (S.R.)
5. Metric space setting II, [AGS08, 2-3], [Mie23, 4] (compactness, topology, GMM). Aim: Understand [AGS08, Theorem 2.3.8] and some aspects of [AGS08, Theorem 2.4.15]. (L.S.)
6. Geometry: Otto view of Wasserstein [Ott01]. Aim: Diffeomorphisms and the induced metric. (B.V.)
7. The description in [Pel14] (the examples and the process space). (T.S.)
8. Tools of measure theory [AGS08, 5]. (J.Z.)
9. Wasserstein distance [AGS08, 7]. Aim: Definitions and properties (not the relation to evolutions). (J.F.)
10. Continuity equation [AGS08, 8]. Aim: Understand the Benamou-Brenier formula. (C.I.)
11. Gradient flows in probability spaces [AGS08, 11]. Aim: Putting it together. (D.W.)

References

- [AGS08] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. *Gradient flows in metric spaces and in the space of probability measures*. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, second edition, 2008.
- [Mie23] Alexander Mielke. An introduction to the analysis of gradient systems, 2023.
- [Ott01] Felix Otto. The geometry of dissipative evolution equations: the porous medium equation. *Comm. Partial Differential Equations*, 26(1-2):101–174, 2001.
- [Pel14] Mark A. Peletier. Variational modelling: Energies, gradient flows, and large deviations, 2014.

Dates

We meet on Tuesdays from 13:00 to 17:00, on the days: **16.4., 23.4., 30.4., 7.5., 14.5.**

We try to cover the material of slightly more than two talks on each of these days.